Derivatives and Integrals of Trig Funcions

Listed in terms of matching

1. $\frac{d(\sin u)}{dx} = (\cos u)\, u'$	⟺	1. $\int (\cos u)\, u' = \sin u + C$
2. $\frac{d(\cos u)}{dx} = (-\sin u)\, u'$	⟺	2. $\int (\sin u)\, u' = -\cos u + C$
3. $\frac{d(\tan u)}{dx} = (\sec^2 u)\, u'$	⟺	3. $\int (\sec^2 u)\, u' = \tan u + C$
4. $\frac{d(\cot u)}{dx} = (-\csc^2 u)\, u'$	⟺	4. $\int (\csc^2 u)\, u' = -\cot u + C$
5. $\frac{d(\sec u)}{dx} = (\sec u \tan u)\, u'$	⟺	5. $\int (\sec u \tan u)\, u' = \sec u + C$
6. $\frac{d(\csc u)}{dx} = (-\csc u \cot u)\, u'$	⟺	6. $\int (\csc u \cot u)\, u' = -\csc u + C$
		7. $\int (\tan u)\, u' = \ln \lvert \sec u \rvert + C$
		8. $\int (\cot u)\, u' = \ln \lvert \sin u \rvert + C$
		9. $\int (\sec u)\, u' = \ln \lvert \sec u + \tan u \rvert + C$
		10. $\int (\csc u)\, u' = \ln \lvert \csc u - \cot u \rvert + C$
11. $\frac{d(\sin^{-1} u)}{dx} = \frac{u'}{\sqrt{1-u^2}}, \ \lvert u \rvert < 1$	⟺	11. $\int \frac{u'}{\sqrt{1-u^2}} = \sin^{-1} u + C, \ \lvert u \rvert < 1$
12. $\frac{d(\cos^{-1} u)}{dx} = \frac{-u'}{\sqrt{1-u^2}}, \ \lvert u \rvert < 1$	⟺	12. $\int \frac{u'}{1+u^2} = \tan^{-1} u + C$
13. $\frac{d(\tan^{-1} u)}{dx} = \frac{u'}{1+u^2}, \ -\infty < u < \infty$	⟺	13. $\int \frac{u'}{u\sqrt{u^2-1}} = \sec^{-1} u + C, \ \lvert u \rvert \geq 1$
14. $\frac{d(\cot^{-1} u)}{dx} = \frac{-u'}{1+u^2}, \ -\infty < u < \infty$		
15. $\frac{d(\sec^{-1} u)}{dx} = \frac{u'}{\lvert u \rvert \sqrt{u^2-1}}, \ \lvert u \rvert > 1$		
16. $\frac{d(\csc^{-1} u)}{dx} = \frac{-u'}{\lvert u \rvert \sqrt{u^2-1}}, \ \lvert u \rvert > 1$		
17. $\frac{d(\ln u)}{dx} = \frac{u'}{u}$	⟺	17. $\int \frac{u'}{u} = \ln \lvert u \rvert + C$
18. $\frac{d(e^u)}{dx} = (e^u)\, u'$	⟺	18. $\int (e^u)\, u' = e^u + C$
19. $\frac{d(a^u)}{dx} = (a^u)\, u' \ln a$	⟺	19. $\int a^u\, u' = \frac{a^u}{\ln a} + C$
20. $\frac{d(\log_a u)}{dx} = \frac{u'}{u \ln a}$		
21. $\frac{d(f^{-1}[y])}{dx} = \frac{1}{f'(x)}$		

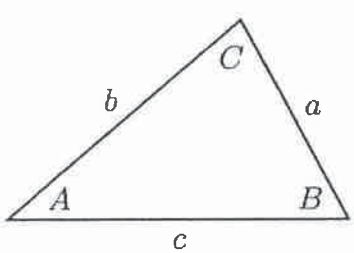
Other Rule: Linear Approximation: $y = f(a) + f'(a)(x - a)$

Angle θ Radians	Angle θ Degrees	sinθ	cosθ	tanθ	cotθ	secθ	cscθ
0	0	0	1	0	- *asymp.*	1	- *asymp.*
$\pi/6$	30	$1/2$	$\sqrt{3}/2$	$1/\sqrt{3}$	$\sqrt{3}$	$2/\sqrt{3}$	2
$\pi/4$	45	$\sqrt{2}/2$	$\sqrt{2}/2$	1	1	$\sqrt{2}$	$\sqrt{2}$
$\pi/3$	60	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$	$1/\sqrt{3}$	2	$2/\sqrt{3}$
$\pi/2$	90	1	0	- *asymp.*	0	- *asymp.*	1
$2\pi/3$	120	$\sqrt{3}/2$	$-1/2$	$-\sqrt{3}$	$-1/\sqrt{3}$	-2	$2/\sqrt{3}$
$3\pi/4$	135	$\sqrt{2}/2$	$-\sqrt{2}/2$	-1	-1	$-\sqrt{2}$	$\sqrt{2}$
$5\pi/6$	150	$1/2$	$-\sqrt{3}/2$	$-1/\sqrt{3}$	$-\sqrt{3}$	$-2/\sqrt{3}$	2
π	180	0	-1	0	- *asymp.*	-1	- *asymp.*
$7\pi/6$	210	$-1/2$	$-\sqrt{3}/2$	$1/\sqrt{3}$	$\sqrt{3}$	$-2/\sqrt{3}$	-2
$5\pi/4$	225	$-\sqrt{2}/2$	$-\sqrt{2}/2$	1	1	$-\sqrt{2}$	$-\sqrt{2}$
$4\pi/3$	240	$-\sqrt{3}/2$	$-1/2$	$\sqrt{3}$	$1/\sqrt{3}$	-2	$-2/\sqrt{3}$
$3\pi/2$	270	-1	0	- *asymp.*	0	- *asymp.*	-1
$5\pi/3$	300	$-\sqrt{3}/2$	$1/2$	$-\sqrt{3}$	$-1/\sqrt{3}$	2	$-2/\sqrt{3}$
$7\pi/4$	315	$-\sqrt{2}/2$	$\sqrt{2}/2$	-1	-1	$\sqrt{2}$	$-\sqrt{2}$
$11\pi/6$	330	$-1/2$	$\sqrt{3}/2$	$-1/\sqrt{3}$	$-\sqrt{3}$	$2/\sqrt{3}$	-2

Oblique (non – 90°) Triangles

Law of Sines

$$\frac{\text{Sin}A}{a} = \frac{\text{Sin}B}{b} = \frac{\text{Sin}C}{c}$$



Law of Cosines

$$\boldsymbol{c^2 = a^2 + b^2 - 2ab \cdot cosC}$$

$$\boldsymbol{b^2 = a^2 + c^2 - 2ac \cdot cosB}$$

$$\boldsymbol{a^2 = b^2 + c^2 - 2bc \cdot cosA}$$

Area of a Triangle

$$\text{Area} = \frac{1}{2}\text{ab sinC}$$

$$\text{Area} = \frac{1}{2}\text{bc sinA}$$

$$\text{Area} = \frac{1}{2}\text{ac sinB}$$

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\text{where s} = \frac{1}{2}(a+b+c)$$

$\sin x = \frac{1}{\csc x}$

$\cos x = \frac{1}{\sec x}$

$\tan x = \frac{1}{\cot x} = \frac{\sin x}{\cos x}$

degrees → radians: multiply $\frac{\pi}{180}$

radians → degrees: multiply $\frac{180}{\pi}$

$\sin x \to x, 180 - x$

$\cos x \to x, 360 - x$

$\tan x \to x, 180 + x$

$\sin(-x) = -\sin x$

$\cos(-x) = \cos x$

Graphing

$y = a\sin(bx - c) + d$

$y = a\cos(bx - c) + d$

a = amplitude (multiply y − values by a)

period $= \frac{2\pi}{|b|}$ (divide x − values by b)

$-\frac{c}{b}$ = horizontal shift (phase shift)

d = vertical shift (+→ up, −→ down)

$\sin x + \sin y = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$

$\sin x - \sin y = 2\sin\left(\frac{x-y}{2}\right)\cos\left(\frac{x+y}{2}\right)$

$\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$

$\cos x - \cos y = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$

$\sin\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1-\cos x}{2}}$

$\cos\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1+\cos x}{2}}$

$\tan\left(\frac{x}{2}\right) = \frac{1-\cos x}{\sin x} = \frac{\sin x}{1+\cos x} = \pm\sqrt{\frac{1-\cos x}{1+\cos x}}$

Sign depends upon quadrant in which $\frac{x}{2}$ is found.

$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$

$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$

$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$

$\sin kx = 2\sin(k-1)x\cos x - \sin(k-2)x$

$\cos kx = 2\cos(k-1)x\cos x - \cos(k-2)x$

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \csc^2 x$

$\sin 2x = 2\sin x\cos x$

$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$

$\cos 2x = 2\cos^2 x - 1$

$= 1 - 2\sin^2 x$

$= \cos^2 x - \sin^2 x$

$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$

$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$

$\sin x \sin y = \frac{1}{2}[\cos(x-y) - \cos(x+y)]$

$\cos x \cos y = \frac{1}{2}[\cos(x-y) + \cos(x+y)]$

$\sin x \cos y = \frac{1}{2}[\sin(x+y) + \sin(x-y)]$