9.4 Graphing Sine and Cosine Functions

Essential Question What are the characteristics of the graphs of the sine and cosine functions?

1 EXPLORATION: Graphing the Sine Function

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner.

from the unit circle

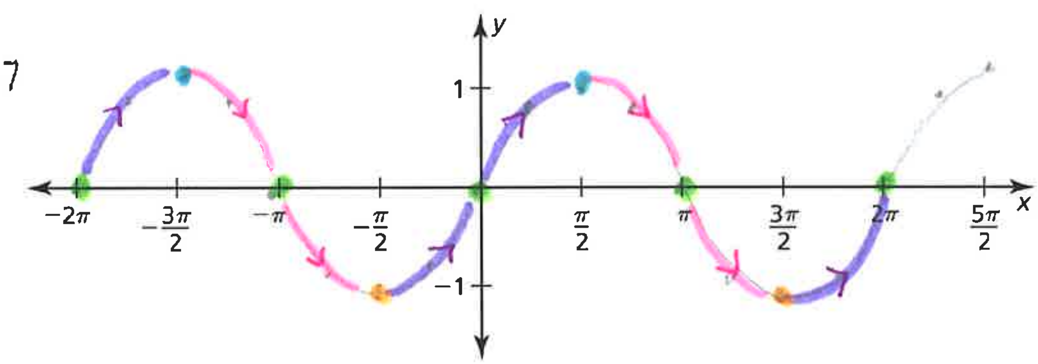
a. Complete the table for $y = \sin x$ where x is an angle measure in radians.

rember $\sin = \frac{y}{r}$

x	-2π	$-\frac{7\pi}{4}$	$-\frac{3\pi}{2}$	$-\frac{5\pi}{4}$	$-\pi$	$-\frac{3\pi}{4}$	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0
$y = \sin x$	0	$\frac{\sqrt{2}}{2}$	1	$\frac{\sqrt{2}}{2}$	0	$-\frac{\sqrt{2}}{2}$	-1	$-\frac{\sqrt{2}}{2}$	0
x	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π	$\frac{9\pi}{4}$
$y = \sin x$	$\frac{\sqrt{2}}{2}$	1	$\frac{\sqrt{2}}{2}$	0	$-\frac{\sqrt{2}}{2}$	-1	$-\frac{\sqrt{2}}{2}$	0	$\frac{\sqrt{2}}{2}$

b. Plot the points (x, y) from part (a). Draw a smooth curve through the points to sketch the graph of $y = \sin x$.

$\frac{\sqrt{2}}{2} \approx .707$



c. Use the graph to identify the x-intercepts, the x-values where the local maximums and minimums occur, and the intervals for which the function is increasing or decreasing over $-2\pi \le x \le 2\pi$

x-intercepts	local max	local min	increasing	decreasing
-2π	$-\frac{3\pi}{2}$	$-\frac{\pi}{2}$	$-2\pi < x < -\frac{3\pi}{2}$	$-\frac{3\pi}{2} < x < -\frac{\pi}{2}$
$-\pi$			$-\frac{\pi}{2} < x < \frac{\pi}{2}$	
0	$\frac{\pi}{2}$	$\frac{3\pi}{2}$		$\frac{\pi}{2} < x < \frac{3\pi}{2}$
π			$\frac{3\pi}{2} < x < 2\pi$	
2π				

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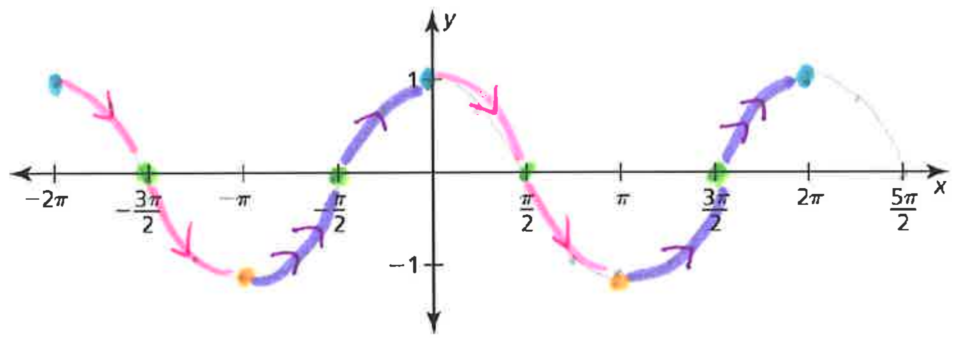
2 **EXPLORATION:** Graphing the Cosine Function

a. Complete the table for $y = \cos x$ using the UNIT CIRCLE

remember $\cos = \frac{x}{r}$

x	-2π	$-\frac{7\pi}{4}$	$-\frac{3\pi}{2}$	$-\frac{5\pi}{4}$	$-\pi$	$-\frac{3\pi}{4}$	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0
$y = \cos x$	1	$\frac{\sqrt{2}}{2}$	0	$-\frac{\sqrt{2}}{2}$	-1	$-\frac{\sqrt{2}}{2}$	0	$\frac{\sqrt{2}}{2}$	1
x	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π	$\frac{9\pi}{4}$
$y = \cos x$	$\frac{\sqrt{2}}{2}$	0	$-\frac{\sqrt{2}}{2}$	-1	$-\frac{\sqrt{2}}{2}$	0	$\frac{\sqrt{2}}{2}$	1	$\frac{\sqrt{2}}{2}$

b. Plot the points (x, y) from part (a) and sketch the graph of $y = \cos x$



c. Use the graph to identify the x-intercepts, the x-values where the local maximums and minimums occur, and the intervals for which the function is increasing or decreasing over $-2\pi \le x \le 2\pi$

x-intercept	local max	local min	increasing	decreasing
$-\frac{3\pi}{2}$	-2π	$-\pi$	$-\pi < x < 0$	$-2\pi < x < -\pi$
$-\frac{\pi}{2}$	0	π	$\pi < x < 2\pi$	$0 < x < \pi$
$\frac{\pi}{2}$	2π			
$\frac{3\pi}{2}$				

Communicate Your Answer

3. What are the characteristics of the graphs of the sine and cosine functions?

min/max = same   repeating pattern

4. Describe the end behavior of the graph of $y = \sin x$.

oscillates b/w 1 and −1

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In your own words, write the meaning of each vocabulary term.

amplitude

high or low

periodic function

regularly

cycle

how long does it take to repeat

period

time frame of a cycle

phase shift

transformation rule

midline

middle of.

Core Concepts

Characteristics of $y = \cos x$ and $y = \sin x$

- The domain of each function is all real numbers.
- The range of each function is $-1 \le y \le 1$. So, the minimum value = -1 and the maximum value = 1.
- The **amplitude** of the graph of each function is one-half of the difference of the maximum value and the minimum value, or $\frac{1}{2}[1 - (-1)] = 1$.

  $\frac{\text{highest} - \text{lowest}}{2}$

- Each function is **periodic**, which means that its graph has a repeating pattern. The shortest repeating portion of the graph is called a **cycle**. The horizontal length of each cycle is called the **period**. The graph of each function has a period of 2π.

  period

- The x-intercepts for $y = \sin x$ occur when $x = 0, \pm\pi, \pm2\pi, \pm3\pi, \ldots$.

  $+\pi$ $+\pi$

- The x-intercepts for $y = \cos x$ occur when $x = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2}, \ldots$.

  $+\pi$ $+\pi$

  Since

  $\frac{\pi}{2} + \frac{2\pi}{2} = \frac{3\pi}{2}$

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Amplitude and Period

The amplitude and period of the graphs of $y = a \sin bx$ and $y = a \cos bx$ where $a \neq 0$ and $b \neq 0$

Amplitude $= |a|$

distance is what matters ...not the direction/flipping

Period $= \dfrac{2\pi}{|b|}$

again the size is what matters. not the +/− to flip

Graphing $y = a \sin b(x - h) + k$ and $y = a \cos b(x - h) + k$

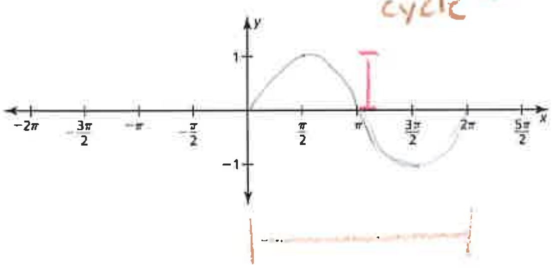
where $a > 0$ and $b > 0$, follow these steps:

Step 1 Identify the amplitude a, the period $\dfrac{2\pi}{b}$, the horizontal shift h, and the vertical shift k of the graph.

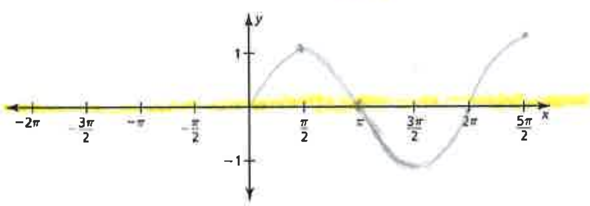
or cycle

(+) left
(−) right

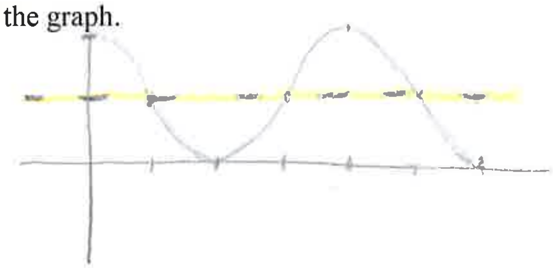
(+) up
(−) down



Step 2 Draw the horizontal line $y = k$ called the **midline** of the graph.

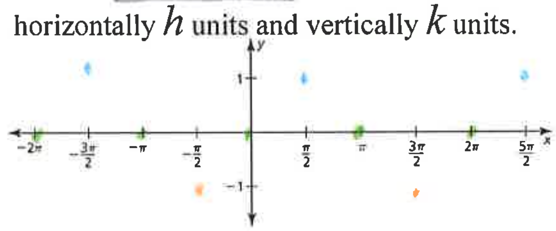


or



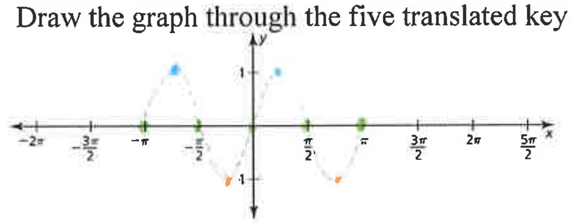
$+1 = k$

Step 3 Find the five key points by translating the key points of $y = a \sin bx$ or $y = a \cos bx$ horizontally h units and vertically k units.



highest
middle
lowest

Step 4 Draw the graph through the five translated key points.



* if your using the transformation concepts, you do not need to use a calculator to graph

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Extra Practice

In Exercises 1–4, identify the amplitude and period of the function. Then graph the function and describe the graph of *g* as a transformation of the graph of its parent function.

1. $g(x) = \sin 2x$

nothing, so automatically 1

we need to say shrink of the period

amplitude = 1 and period = $\frac{2\pi}{2} = \pi$

(a much shorter cycle than the standard)

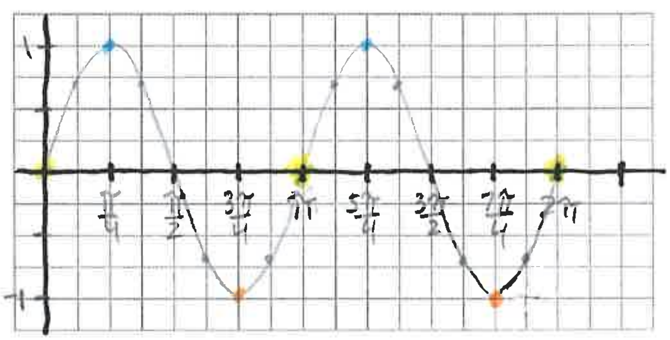
start w/ the original sine table

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
sin	0		1		0		−1		0

cycle

then apply all the changes

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
$g(x)$	0	1	0	−1	0	1	0	−1	0



2. $g(x) = \frac{1}{3}\cos 2x$

amplitude = $\frac{1}{3}$ so shrink .. shorter

period = $\frac{2\pi}{2} = \pi$ (shrink of the cycle) .... squished in

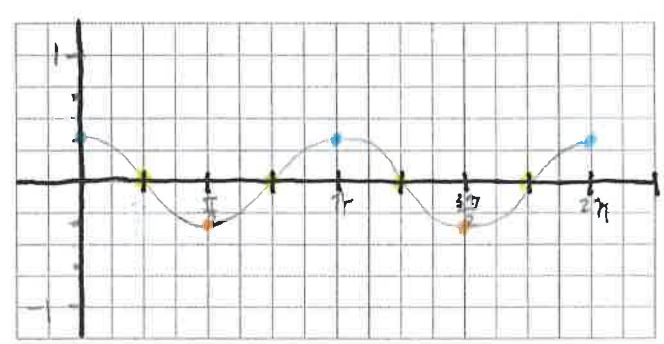
original cosine table

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
cos	1		0		−1		0		1

cycle

then apply the changes

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
$g(x)$	$\frac{1}{3}$	0	$-\frac{1}{3}$	0	$\frac{1}{3}$	0	$-\frac{1}{3}$	0	$\frac{1}{3}$



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In Exercises 1–4, identify the amplitude and period of the function. Then graph the function and describe the graph of *g* as a transformation of the graph of its parent function.

3. $g(x) = 4 \sin 2\pi x$

amplitude = 4 ... so taller

period = $\frac{2\pi}{2\pi}$ = 1 (notice that the cycle is no longer π) squished in like an accordion

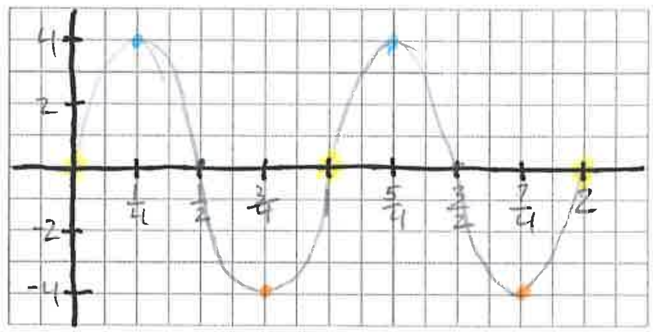
original sine graph

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
sin	0		1		0		-1		0

cycle

the new graph with changes

x	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1	$\frac{5}{4}$	$\frac{3}{2}$	$\frac{7}{4}$	2
g(x)	0	4	0	-4	0	4	0	-4	0



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4. $g(x) = \frac{1}{2} \cos 3\pi x$

amplitude = $\frac{1}{2}$ ... so shorter

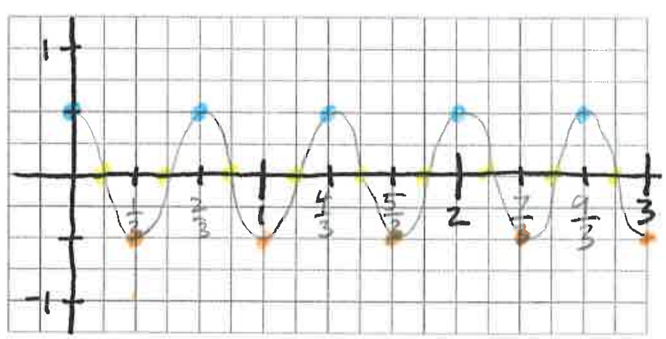
period = $\frac{2\pi}{3\pi}$ = $\frac{2}{3}$ (notice no longer π) squished in again

original cosine table

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
cos	1		0		-1		0		1

cycle

x	0	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{5}{6}$	1	$\frac{7}{6}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{5}{3}$
g(x)	.5	0	-.5	0	.5	0	-.5	0	.5		-.5



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In Exercises 5 and 6, graph the function.

figure out your midline and starting point then your cycle and high/low

5. $g(x) = \sin \frac{1}{2}(x - \pi) + 1$

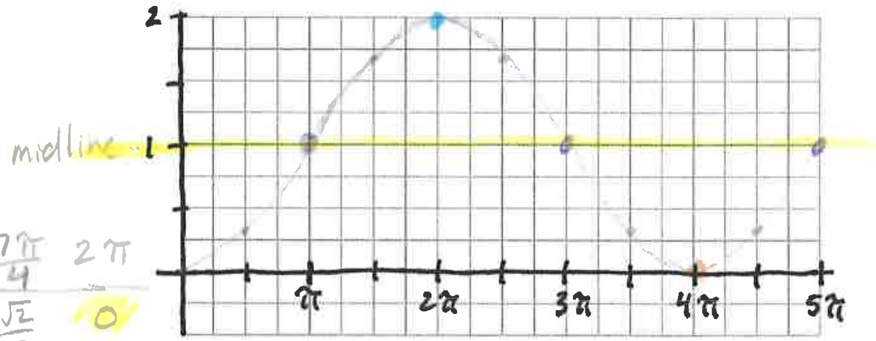
amplitude = 1

period = $\frac{2\pi}{.5} = 4\pi$ .. stretched out

shift right π

up 1

|— cycle —|



original

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
sin	0 start	$\frac{\sqrt{2}}{2}$	1	$\frac{\sqrt{2}}{2}$	0	$-\frac{\sqrt{2}}{2}$	-1	$-\frac{\sqrt{2}}{2}$	0

apply all the changes

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π	$\frac{9\pi}{4}$	$\frac{5\pi}{2}$	$\frac{11\pi}{4}$	3π	$\frac{13\pi}{4}$	$\frac{7\pi}{2}$	$\frac{15\pi}{4}$	4π	$\frac{17\pi}{4}$	$\frac{9\pi}{2}$	$\frac{19\pi}{4}$	5π
g(x)	0				1 start		$\frac{2+\sqrt{2}}{2}$ 1.707		2		$\frac{2+\sqrt{2}}{2}$ 1.707		1		$\frac{2-\sqrt{2}}{2}$ .29		0		$\frac{2-\sqrt{2}}{2}$ .29		1

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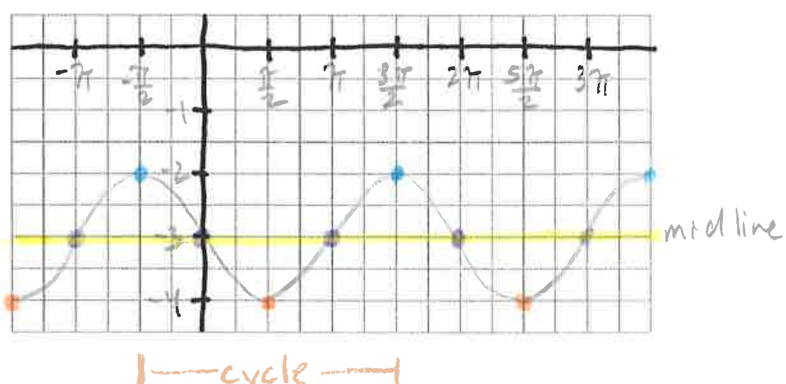
6. $g(x) = \cos\left(x + \frac{\pi}{2}\right) - 3$

amplitude = 1

period = $\frac{2\pi}{1} = 2\pi$ (normal)

shift left $\frac{\pi}{2}$

down 3



original

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
cos	1 start		0		-1		0		1

apply the changes

x	$-\pi$	$-\frac{3\pi}{4}$	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
g(x)	-3		-2 start		-3		-4		-3		-2		-3

← shift