6.5 Properties of Logarithms

In your own words, write the meaning of each vocabulary term.

Base

...... in Exponent Form ... in Log Form

$\boxed{2^3 = 8}$ $\boxed{\log_2 8 = 3}$

properties of exponents

always flip add

$\boxed{2^0 = 1}$ $\boxed{2^{-3} = \frac{1}{2^3} = \frac{1}{8}}$ $\boxed{2^5 \cdot 2^3 = 2^8}$

$\boxed{\frac{2^5}{2^3} = 2^2}$ $\boxed{\frac{2^3}{2^5} = \frac{1}{2^2}}$ $\boxed{(2^3)^5 = 2^{15}}$

Subtract subtract and (+) exponents multiply

Core Concepts

Properties of Logarithms

Let b, m, and n be positive real numbers with $b \neq 1$.

multiply Division Exponent

Product Property **Quotient Property** **Power Property**

$$\log_b (mn) = \log_b m + \log_b n$$

addition

$$\log_b \frac{m}{n} = \log_b m - \log_b n$$

Subtraction

$$\log_b m^n = n \log_b m$$

multiplication

* bases are still the same
** location does matter

Change-of-Base Formula

If a, b, and c are positive real numbers with $b \neq 1$ and $c \neq 1$, then $\log_c a = \dfrac{\log_b a}{\log_b c}$.

In particular, $\log_c a = \dfrac{\log a}{\log c}$ and $\log_c a = \dfrac{\ln a}{\ln c}$.

so if you want to find...

$\log_c a$ ...... write it as $= \dfrac{\log a}{\log c}$

(using a calculator

both bases are now 10 and calculators automatically use log bases of 10

*If you do NOT follow directions... you will <u>not</u> get full credit on the Test!!

6.5 Extra Practice

In Exercises 1–4, use $\log_2 5 \approx 2.322$ and $\log_2 12 \approx 3.585$ to evaluate the logarithm.

1. $\log_2 60$

$= \log_2 (5 \cdot 12)$ product

$= \boxed{\log_2 5} + \boxed{\log_2 12}$

$= 2.322 + 3.585$

$= \boxed{5.907}$ ... use change of base to double check. :)

2. $\log_2 \frac{1}{144} = \log_2 12^{-2}$ power

$= -2\left(\boxed{\log_2 12}\right)$

$= -2(3.585)$

$= \boxed{-7.17}$

3. $\log_2 \frac{12}{25}$ Quotient

$= \log_2 12 - \log_2 25$

$= \log_2 12 - \log_2 (5)^2$ power

$= \boxed{\log_2 12} - 2\boxed{\log_2 5}$

$= 3.585 - 2(2.322)$

$= 3.585 - 4.644 = \boxed{-1.059}$

4. $\log_2 720 = \log_2 (12 \cdot 12 \cdot 5)$

$= \boxed{\log_2 12} + \boxed{\log_2 12} + \boxed{\log_2 5}$

$= 3.585 + 3.585 + 2.322$

$= \boxed{9.492}$

or

$\log_2 12^2 + \log_2 5$

$2 \log_2 12 + \log_2 5 \Rightarrow 2(3.585) + 2.322$

$7.17 + 2.322$

9.492

In Exercises 5–8, expand the logarithmic expression.

5. $\log(10x)$ product

$= \log 10 + \log x$

$= \boxed{1 + \log x}$

when no base is indicated, it's automatically 10

6. $\ln(2x^6)$ product

$= \ln 2 + \ln x^6$ power

$= \boxed{\ln 2 + 6 \ln x}$

7. $\log_3 \frac{x^4}{3y^3}$

$= \log_3 x^4 - \log_3 (3y^3)$

$= \log_3 x^4 - [\log_3 3 + \log_3 y^3]$

Distribute

$= 4\log_3 x - \log_3 3 - 3\log_3 y$

$= \boxed{4\log_3 x - 1 - 3\log_3 y}$

8. $\ln \sqrt[4]{3y^2} = \ln (3y^2)^{1/4}$

$= \frac{1}{4}[\ln(3y^2)]$

$= \frac{1}{4}\ln 3 + \frac{1}{4}\ln y^2$

$= \frac{1}{4}\ln 3 + 2\left[\frac{1}{4}\ln y\right]$

$= \boxed{\frac{1}{4}\ln 3 + \frac{1}{2}\ln y}$

6.5 Follow the Directions!!

In Exercises 9–13, condense the logarithmic expression.

9. $\log_2 3 + \log_2 8$

product

$= \log_2 (3)(8)$

$= \boxed{\log_2 24}$

10. $\log_5 4 - 2\log_5 5$

power

$= \log_5 4 - \log_5 (5)^2$

$= \log_5 4 - \log_5 25$

Quotient

$= \boxed{\log_5\left(\frac{4}{25}\right)}$

* If you cancelled ($\log_5 5$) earier then you can not combine with the other $\log_5$

11. $3\ln 6x + \ln 4y$

power

$= \ln (6x)^3 + \ln 4y$

$6^3 x^3$

$= \ln(216x^3) + \ln(4y)$

product

$= \ln (216x^3)(4y)$

$216 \cdot 4$

$= \boxed{\ln(864x^3y)}$

12. $\log_2 625 - \log_2 125 + \frac{1}{3}\log_2 27$

*work Left → Rule

$= \log_2 \frac{625}{125} + \log_2 (27)^{1/3}$

$= \log_2 5 + \log_2 3$

$= \log_2 (5)(3)$

$= \boxed{\log_2 15}$

You know you did it right when you finish with just one log :)

* Use change of base to double check work

13. $-\log_6 6 - \log_6 2y + 2\log_6 3x$

(-1)

$= \log_6 (6)^{-1} - \log_6 2y + \log_6 (3x)^2$

3^2x^2

flip

$= \log_6 \frac{6^{-1}}{2y} + \log(9x^2)$

$= \log_6 \frac{1}{12y} + \log(9x^2)$

since $\frac{1}{6} \cdot \frac{1}{2y}$

$= \log_6 \left(\frac{1}{12y}\right)(9x^2)$

$= \log_6 \frac{9x^2}{12y}$ reduce

$= \boxed{\log_6\left(\frac{3x^2}{4y}\right)}$

6.5 * make sure to write out the change form !

In Exercises 14–17, use the change-of-base formula to evaluate the logarithm.

14. $\log_3 17$

$= \dfrac{\log 17}{\log 3} = \boxed{2.579}$

15. $\log_9 294$

$= \dfrac{\log 294}{\log 9} = \boxed{2.587}$

16. $\log_7 \dfrac{4}{9}$

$= \dfrac{\log \frac{4}{9}}{\log 7} = \boxed{-.417}$

or -0.417

17. $\log_6 \dfrac{1}{10}$

$= \dfrac{\log \frac{1}{10}}{\log 6} = \boxed{-1.285}$

18. For a sound with intensity I (in watts per square meter), the loudness $L(I)$ of the sound (in decibels) is given by the function $L(I) = 10 \log \dfrac{I}{I_0}$, where I_0 is the intensity of a barely audible sound (about 10^{-12} watts per square meter). The intensity of the sound of a certain children's television show is half the intensity of the adult show that is on before it. By how many decibels does the loudness decrease?

19. Hick's Law states that given n equally probable choices, such as choices on a menu, the average human's reaction time T (in seconds) required to choose from those choices is approximately $T = a + b \bullet \log_2(n + 1)$ where a and b are constants. If $a = 4$ and $b = 1$, how much longer would it take a customer to choose what to eat from a menu of 40 items than from a menu of
10 items?