1.6 Trigonometric Functions

You will be able to analyze trigonometric functions and their inverses algebraically, graphically, and numerically and will be able to model periodic behavior with sinusoids.

- Radian measure
- The six basic trigonometric functions
- Periodicity
- Properties of trigonometric functions (symmetry, period)
- Transformations of trigonometric functions
- Sinusoids and their properties (amplitude, period, frequency, shifts)
- Inverse trigonometric functions and their graphs

Radian Measure

The **radian measure** of the angle ACB at the center of the unit circle (Figure 1.36) equals the length of the arc that ACB cuts from the unit circle.

EXAMPLE 1 Finding Arc Length

Find the length of an arc subtended on a circle of radius 3 by a central angle of measure $2\pi/3$.

SOLUTION

According to Figure 1.36, if s is the length of the arc, then

$$s = r\theta = 3(2\pi/3) = 2\pi.$$

Now Try Exercise 1.

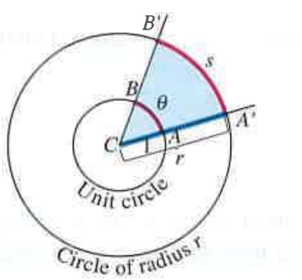


Figure 1.36 The radian measure of angle ACB is the length θ of arc AB on the unit circle centered at C. The value of θ can be found from any other circle, however, as the ratio s/r.

When an angle of measure θ is placed in *standard position* at the center of a circle of radius r (Figure 1.37), the six basic trigonometric functions of θ are defined as follows:

$$\textbf{sine: } \sin\theta = \frac{y}{r} \qquad \textbf{cosecant: } \csc\theta = \frac{r}{y}$$

$$\textbf{cosine: } \cos\theta = \frac{x}{r} \qquad \textbf{secant: } \sec\theta = \frac{r}{x}$$

$$\textbf{tangent: } \tan\theta = \frac{y}{x} \qquad \textbf{cotangent: } \cot\theta = \frac{x}{y}$$

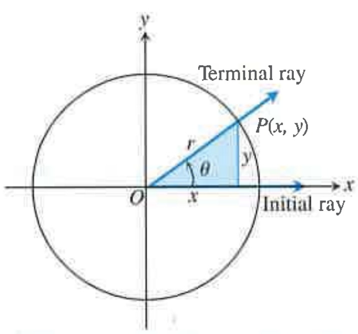


Figure 1.37 An angle θ in standard position.

Graphs of Trigonometric Functions

When we graph trigonometric functions in the coordinate plane, we usually denote the independent variable (radians) by x instead of θ. Figure 1.38 on the next page shows sketches of the six trigonometric functions. It is a good exercise for you to compare these with what you see in a grapher viewing window. (Some graphers have a "trig viewing window.")

EXPLORATION 1 Unwrapping Trigonometric Functions

Set your grapher in *radian mode*, parametric mode, and *simultaneous mode* (all three). Enter the parametric equations

$$x_1 = \cos t, \quad y_1 = \sin t \quad \text{and} \quad x_2 = t, \quad y_2 = \sin t.$$

1. Graph for $0 \le t \le 2\pi$ in the window $[-1.5, 2\pi]$ by $[-2.5, 2.5]$. Describe the two curves. (You may wish to make the viewing window square.)
2. Use TRACE to compare the y values of the two curves.
3. Repeat part 2 in the window $[-1.5, 4\pi]$ by $[-5, 5]$, using the parameter interval $0 \le t \le 4\pi$.
4. Let $y_2 = \cos t$. Use TRACE to compare the x values of curve 1 (the unit circle) with the y values of curve 2 using the parameter intervals $[0, 2\pi]$ and $[0, 4\pi]$.
5. Set $y_2 = \tan t$, $\csc t$, $\sec t$, and $\cot t$. Graph each in the window $[-1.5, 2\pi]$ by $[-2.5, 2.5]$ using the interval $0 \le t \le 2\pi$. How is a y value of curve 2 related to the corresponding point on curve 1? (Use TRACE to explore the curves.)

Angle Convention: Use Radians

From now on in this book it is assumed that all angles are measured in radians unless degrees or some other unit is stated explicitly. When we talk about the angle $\pi/3$, we mean $\pi/3$ radians (which is 60°), not $\pi/3$ degrees. When you do calculus, keep your calculator in radian mode.

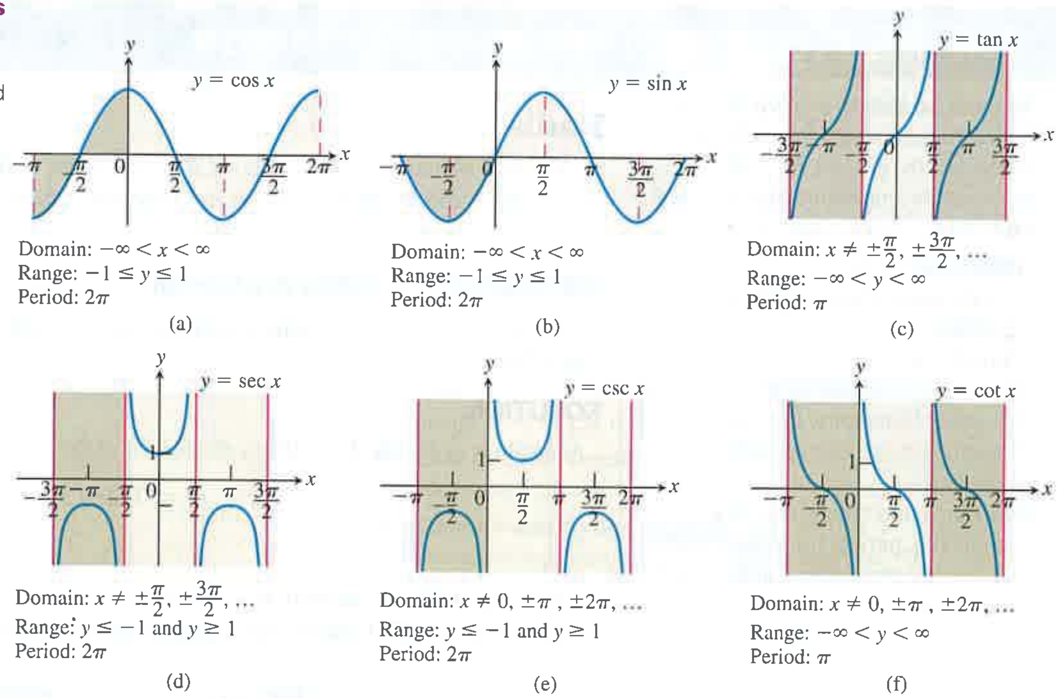


Figure 1.38 Graphs of the (a) cosine, (b) sine, (c) tangent, (d) secant, (e) cosecant, and (f) cotangent functions using radian measure.

Periods of Trigonometric Functions

Period π: $\tan(x + \pi) = \tan x$
$\cot(x + \pi) = \cot x$

Period 2π: $\sin(x + 2\pi) = \sin x$
$\cos(x + 2\pi) = \cos x$
$\sec(x + 2\pi) = \sec x$
$\csc(x + 2\pi) = \csc x$

Periodicity

When an angle of measure θ and an angle of measure $\theta + 2\pi$ are in standard position, their terminal rays coincide. The two angles therefore have the same trigonometric function values:

$$\cos(\theta + 2\pi) = \cos\theta \quad \sin(\theta + 2\pi) = \sin\theta \quad \tan(\theta + 2\pi) = \tan\theta$$
$$\sec(\theta + 2\pi) = \sec\theta \quad \csc(\theta + 2\pi) = \csc\theta \quad \cot(\theta + 2\pi) = \cot\theta \tag{1}$$

Similarly, $\cos(\theta - 2\pi) = \cos\theta$, $\sin(\theta - 2\pi) = \sin\theta$, and so on.

We see the values of the trigonometric functions repeat at regular intervals. We describe this behavior by saying that the six basic trigonometric functions are *periodic*.

DEFINITION Periodic Function, Period

A function $f(x)$ is **periodic** if there is a positive number p such that $f(x + p) = f(x)$ for every value of x. The smallest such value of p is the **period** of f.

As we can see in Figure 1.38, the functions $\cos x$, $\sin x$, $\sec x$, and $\csc x$ are periodic with period 2π. The functions $\tan x$ and $\cot x$ are periodic with period π.

Even and Odd Trigonometric Functions

The graphs in Figure 1.38 suggest that $\cos x$ and $\sec x$ are even functions because their graphs are symmetric about the y-axis. The other four basic trigonometric functions are odd.

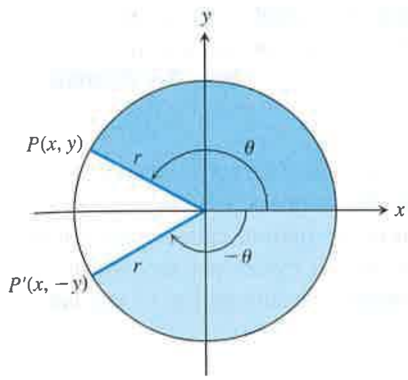


Figure 1.39 Angles of opposite sign. (Example 2)

EXAMPLE 2 Confirming Even and Odd

Show that cosine is an even function and sine is odd.

SOLUTION

From Figure 1.39 it follows that

$$\cos(-\theta) = \frac{x}{r} = \cos\theta, \quad \sin(-\theta) = \frac{-y}{r} = -\sin\theta,$$

so cosine is an even function and sine is odd. ***Now Try Exercise 5.***

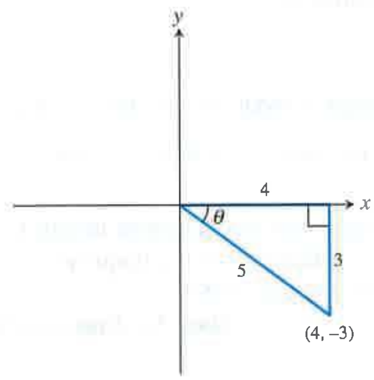


Figure 1.40 The angle θ in standard position. (Example 3)

EXAMPLE 3 Finding Trigonometric Values

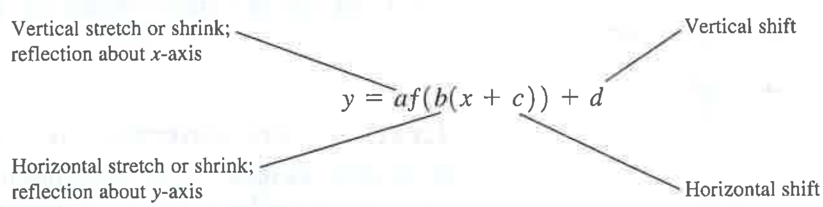
Find all the trigonometric values of θ if $\sin\theta = -3/5$ and $\tan\theta < 0$.

SOLUTION

The angle θ is in the fourth quadrant, as shown in Figure 1.40, because its sine and tangent are negative. From this figure we can read that $\cos\theta = 4/5$, $\tan\theta = -3/4$, $\csc\theta = -5/3$, $\sec\theta = 5/4$, and $\cot\theta = -4/3$. ***Now Try Exercise 9.***

Transformations of Trigonometric Graphs

The rules for shifting, stretching, shrinking, and reflecting the graph of a function apply to the trigonometric functions. The following diagram will remind you of the controlling parameters.



The general sine function, or **sinusoid,** can be written in the form

$$f(x) = A\sin[B(x - C)] + D,$$

where $|A|$ is the *amplitude,* $|2\pi/B|$ is the *period,* C is the *horizontal shift,* and D is the *vertical shift.*

EXAMPLE 4 Graphing a Trigonometric Function

Determine the **(a)** period, **(b)** domain, **(c)** range, and **(d)** draw the graph of the function $y = 3\cos(2x - \pi) + 1$.

SOLUTION

We can rewrite the function in the form

$$y = 3\cos\left[2\left(x - \frac{\pi}{2}\right)\right] + 1.$$

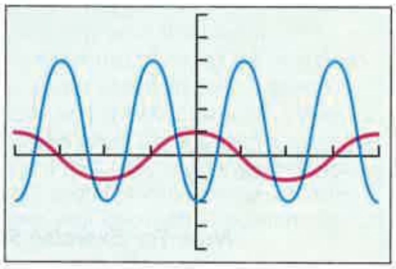
(a) The period is given by $|2\pi/B|$, where $B = 2$. The period is π.

(b) The domain is $(-\infty, \infty)$.

(c) The graph is a basic cosine curve with amplitude 3 that has been shifted up 1 unit. Thus, the range is $[-2, 4]$.

continued

$y = 3\cos(2x - \pi) + 1$, $y = \cos x$



$[-2\pi, 2\pi]$ by $[-4, 6]$

Figure 1.41 The graph of $y = 3\cos(2x - \pi) + 1$ (blue) and the graph of $y = \cos x$ (red). (Example 4)

(d) The graph has been shifted to the right $\pi/2$ units. The graph is shown in Figure 1.41 together with the graph of $y = \cos x$. Notice that four periods of $y = 3\cos(2x - \pi) + 1$ are drawn in this window. ***Now Try Exercise 13.***

Musical notes are produced by pressure waves in the air. The wave behavior can be modeled by sinusoids in which the amplitude affects the loudness and the period affects the tone we hear. In this context, it is the reciprocal of the period, called the *frequency,* that is used to describe the tone. We measure frequency in cycles per second, or hertz (1 Hz = 1 cycle per second), so in this context we would measure period in seconds per cycle.

EXAMPLE 5 Finding the Frequency of a Musical Note

A computer analyzes the pressure displacement versus time for the wave produced by a tuning fork and gives its equation as $y = 0.6\sin(2488.6x - 2.832) + 0.266$. Estimate the frequency of the note produced by the tuning fork.

SOLUTION

The period is $\dfrac{2\pi}{2488.6}$, so the frequency is $\dfrac{2488.6}{2\pi}$, which is about 396 Hz. (Notice that the amplitude, horizontal shift, and vertical shift are not important for determining the frequency of the note.)

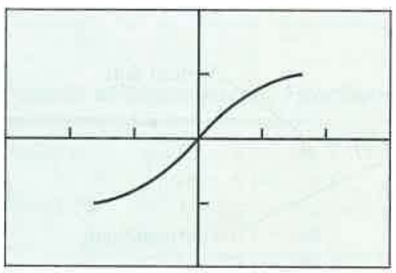
A tuning fork vibrating at a frequency of 396 Hz produces the note G above middle C on the "pure tone" scale. It is a few cycles per second different from the frequency of the G we hear on a piano's "tempered" scale, which is 392 Hz.

Now Try Exercise 23.

Inverse Trigonometric Functions

None of the six basic trigonometric functions graphed in Figure 1.38 is one-to-one. These functions do not have inverses. However, in each case the domain can be restricted to produce a new function that does have an inverse, as illustrated in Example 6.

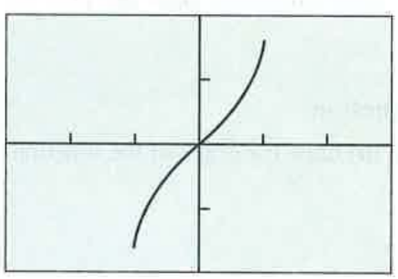
$x = t$, $y = \sin t$, $-\frac{\pi}{2} \le t \le \frac{\pi}{2}$



$[-3, 3]$ by $[-2, 2]$

(a)

$x = \sin t$, $y = t$, $-\frac{\pi}{2} \le t \le \frac{\pi}{2}$



$[-3, 3]$ by $[-2, 2]$

(b)

Figure 1.42 (a) A restricted sine function and (b) its inverse. (Example 6)

EXAMPLE 6 Restricting the Domain of the Sine Function

Show that the function $y = \sin x$, $-\pi/2 \le x \le \pi/2$, is one-to-one, and graph its inverse.

SOLUTION

Figure 1.42a shows the graph of this restricted sine function using the parametric equations

$$x_1 = t, \quad y_1 = \sin t, \quad -\frac{\pi}{2} \le t \le \frac{\pi}{2}.$$

This restricted sine function is one-to-one because it does not repeat any output values. It therefore has an inverse, which we graph in Figure 1.42b by interchanging the ordered pairs using the parametric equations

$$x_2 = \sin t, \quad y_2 = t, \quad -\frac{\pi}{2} \le t \le \frac{\pi}{2}.$$

Now Try Exercise 25.

The inverse of the restricted sine function of Example 6 is called the *inverse sine function.* The inverse sine of x is the angle whose sine is x. It is denoted by $\sin^{-1} x$ or arcsin x. Either notation is read "arcsine of x" or "the inverse sine of x."

The domains of the other basic trigonometric functions can also be restricted to produce a function with an inverse. The domains and ranges of the resulting inverse functions become parts of their definitions.

DEFINITIONS Inverse Trigonometric Functions

Function	Domain	Range
$y = \cos^{-1} x$	$-1 \le x \le 1$	$0 \le y \le \pi$
$y = \sin^{-1} x$	$-1 \le x \le 1$	$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$
$y = \tan^{-1} x$	$-\infty < x < \infty$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$
$y = \sec^{-1} x$	$\lvert x \rvert \ge 1$	$0 \le y \le \pi, y \ne \frac{\pi}{2}$
$y = \csc^{-1} x$	$\lvert x \rvert \ge 1$	$-\frac{\pi}{2} \le y \le \frac{\pi}{2}, y \ne 0$
$y = \cot^{-1} x$	$-\infty < x < \infty$	$0 < y < \pi$

The graphs of the six inverse trigonometric functions are shown in Figure 1.43.

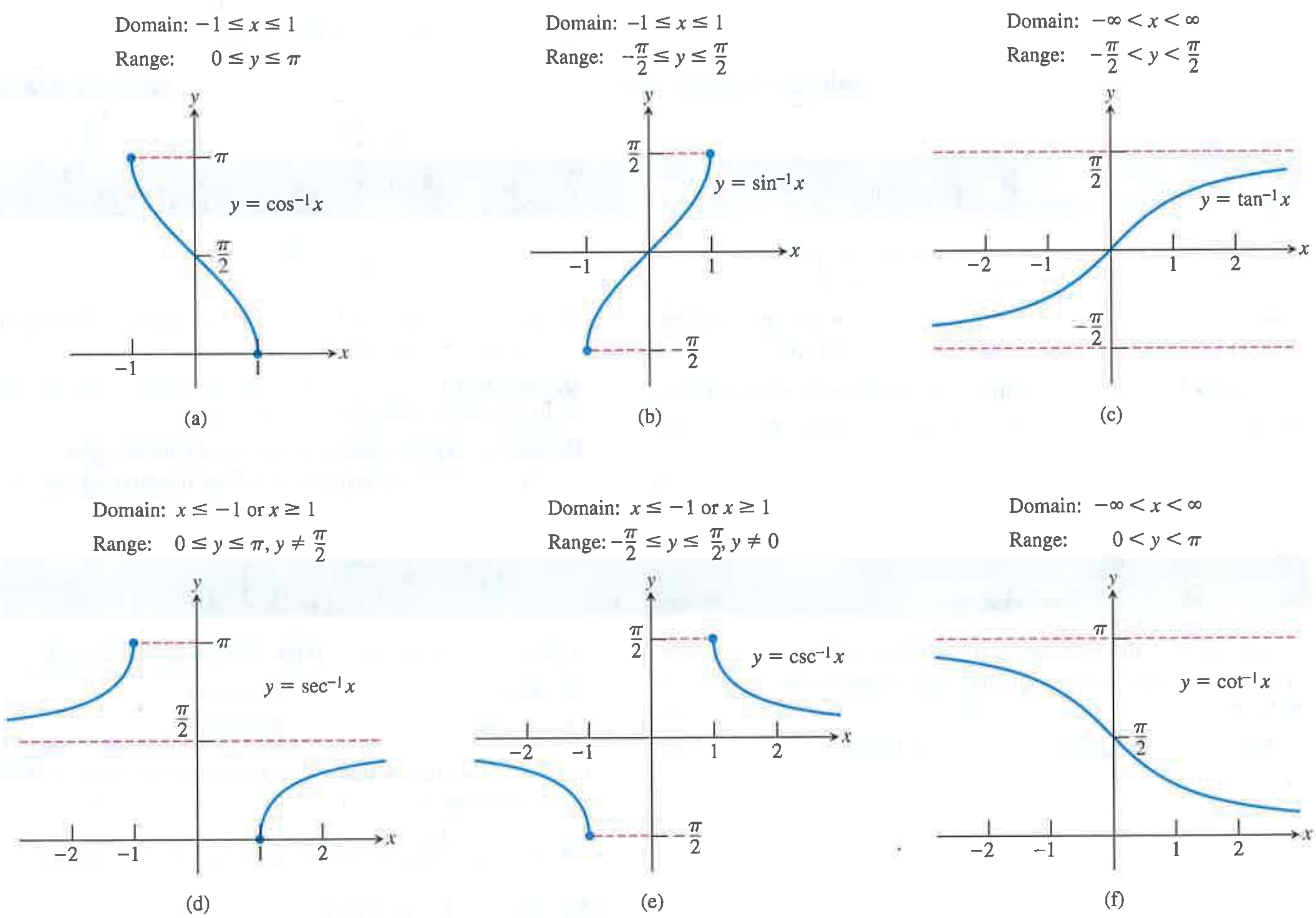


Figure 1.43 Graphs of (a) $y = \cos^{-1} x$, (b) $y = \sin^{-1} x$, (c) $y = \tan^{-1} x$, (d) $y = \sec^{-1} x$, (e) $y = \csc^{-1} x$, and (f) $y = \cot^{-1} x$.

EXAMPLE 7 Finding Angles in Degrees and Radians

Find the measure of $\cos^{-1}(-0.5)$ in degrees and radians.

SOLUTION

Put the calculator in degree mode and enter $\cos^{-1}(-0.5)$. The calculator returns 120, which means 120 degrees. Now put the calculator in radian mode and enter $\cos^{-1}(-0.5)$. The calculator returns 2.094395102, which is the measure of the angle in radians. You can check that $2\pi/3 \approx 2.094395102$. *Now Try Exercise 27.*

EXAMPLE 8 Using the Inverse Trigonometric Functions

Solve for x.

(a) $\sin x = 0.7$ in $0 \le x < 2\pi$

(b) $\tan x = -2$ in $-\infty < x < \infty$

SOLUTION

(a) Notice that $x = \sin^{-1}(0.7) \approx 0.775$ is in the first quadrant, so 0.775 is one solution of this equation. The angle $\pi - x$ is in the second quadrant and has sine equal to 0.7. Thus two solutions in this interval are

$$\sin^{-1}(0.7) \approx 0.775 \quad \text{and} \quad \pi - \sin^{-1}(0.7) \approx 2.366.$$

(b) The angle $x = \tan^{-1}(-2) \approx -1.107$ is in the fourth quadrant and is the only solution to this equation in the interval $-\pi/2 < x < \pi/2$ where $\tan x$ is one-to-one. Since $\tan x$ is periodic with period π, the solutions to this equation are of the form

$$\tan^{-1}(-2) + k\pi \approx -1.107 + k\pi$$

where k is any integer. *Now Try Exercise 31.*

Quick Review 1.6 *(For help, go to Sections 1.2 and 1.6.)*

Exercise numbers with a gray background indicate problems that the authors have designed to be solved without a calculator.

In Exercises 1–4, convert from radians to degrees or degrees to radians.

1. $\pi/3$ **2.** -2.5 **3.** $-40°$ **4.** $45°$

In Exercises 5–7, solve the equation graphically in the given interval.

5. $\sin x = 0.6, \quad 0 \le x \le 2\pi$ **6.** $\cos x = -0.4, \quad 0 \le x \le 2\pi$

7. $\tan x = 1, \quad -\dfrac{\pi}{2} \le x < \dfrac{3\pi}{2}$

8. Show that $f(x) = 2x^2 - 3$ is an even function. Explain why its graph is symmetric about the y-axis.

9. Show that $f(x) = x^3 - 3x$ is an odd function. Explain why its graph is symmetric about the origin.

10. Give one way to restrict the domain of the function $f(x) = x^4 - 2$ to make the resulting function one-to-one.

Section 1.6 Exercises

In Exercises 1–4, the angle lies at the center of a circle and subtends an arc of the circle. Find the missing angle measure, circle radius, or arc length.

Angle	Radius	Arc Length
1. $5\pi/8$	2	?
2. 175°	?	10
3. ?	14	7
4. ?	6	$3\pi/2$

In Exercises 5–8, determine if the function is even or odd.

5. secant **6.** tangent

7. cosecant **8.** cotangent

In Exercises 9 and 10, find all the trigonometric values of θ with the given conditions.

9. $\cos\theta = -\dfrac{15}{17}, \quad \sin\theta > 0$

10. $\tan\theta = -1, \quad \sin\theta < 0$

In Exercises 11–14, determine **(a)** the period, **(b)** the domain, **(c)** the range, and **(d)** draw the graph of the function.

11. $y = 3\csc(3x + \pi) - 2$ **12.** $y = 2\sin(4x + \pi) + 3$

13. $y = -3\tan(3x + \pi) + 2$

14. $y = 2\sin\left(2x + \dfrac{\pi}{3}\right)$

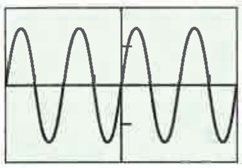
In Exercises 15 and 16, choose an appropriate viewing window to display two complete periods of each trigonometric function in radian mode.

15. **(a)** $y = \sec x$ **(b)** $y = \csc x$ **(c)** $y = \cot x$

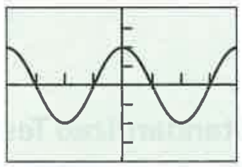
16. **(a)** $y = \sin x$ **(b)** $y = \cos x$ **(c)** $y = \tan x$

In Exercises 17–22, specify **(a)** the period, **(b)** the amplitude, and **(c)** identify the viewing window that is shown. [Caution: Do not assume that the tick marks on both axes are at integer values.]

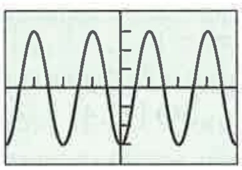
17. $y = 1.5\sin 2x$



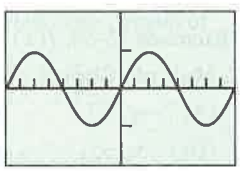
18. $y = 2\cos 3x$



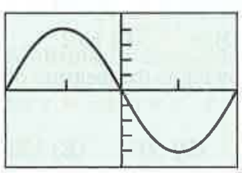
19. $y = -3\cos 2x$



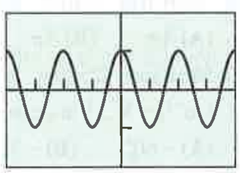
20. $y = 5\sin\dfrac{x}{2}$



21. $y = -4\sin\dfrac{\pi}{3}x$



22. $y = \cos\pi x$



23. The frequencies for the seven "white key" notes produced on the tempered scale of a piano (starting with middle C) are shown in Table 1.4. A computer analyzes the pressure displacement versus time for the wave produced by a tuning fork and gives its equation as $y = 1.23\sin(2073.55x - 0.49) + 0.44$.

(a) Estimate the frequency of the note produced by the tuning fork.

(b) Identify the note produced by the tuning fork.

TABLE 1.4 Frequencies of Musical Notes

C	D	E	F	G	A	B
262	294	330	349	392	440	494

24. ***Temperature Data*** Table 1.5 gives the average monthly temperatures for St. Louis for a 12-month period starting with January. Model the monthly temperature with an equation of the form

$$y = a\sin[b(t - h)] + k,$$

with y in degrees Fahrenheit, t in months, as follows:

TABLE 1.5
Temperature Data for St. Louis

Time (months)	Temperature (°F)
1	34
2	30
3	39
4	44
5	58
6	67
7	78
8	80
9	72
10	63
11	51
12	40

(a) Find the value of b, assuming that the period is 12 months.

(b) How is the amplitude a related to the difference $80° - 30°$?

(c) Use the information in (b) to find k.

(d) Find h, and write an equation for y.

(e) Superimpose a graph of y on a scatter plot of the data.

In Exercises 25–26, show that the function is one-to-one, and graph its inverse.

25. $y = \cos x,\quad 0 \le x \le \pi$ **26.** $y = \tan x,\quad -\dfrac{\pi}{2} < x < \dfrac{\pi}{2}$

In Exercises 27–30, give the measure of the angle in radians and degrees. Give exact answers whenever possible.

27. $\sin^{-1}(0.5)$ **28.** $\sin^{-1}\left(-\dfrac{\sqrt{2}}{2}\right)$

29. $\tan^{-1}(-5)$ **30.** $\cos^{-1}(0.7)$

In Exercises 31–36, solve the equation in the specified interval.

31. $\tan x = 2.5,\quad 0 \le x \le 2\pi$

32. $\cos x = -0.7,\quad 2\pi \le x < 4\pi$

33. $\csc x = 2,\quad 0 < x < 2\pi$ **34.** $\sec x = -3,\quad -\pi \le x < \pi$

35. $\sin x = -0.5,\quad -\infty < x < \infty$

36. $\cot x = -1,\quad -\infty < x < \infty$

In Exercises 37–40, use the given information to find the values of the six trigonometric functions at the angle θ. Give exact answers.

37. $\theta = \sin^{-1}\left(\frac{8}{17}\right)$

38. $\theta = \tan^{-1}\left(-\frac{5}{12}\right)$

39. The point $P(-3, 4)$ is on the terminal side of θ.

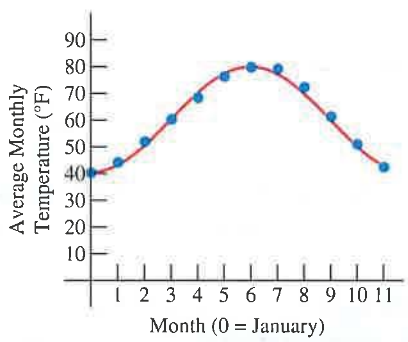
40. The point $P(-2, 2)$ is on the terminal side of θ.

In Exercises 41 and 42, evaluate the expression.

41. $\sin\left(\cos^{-1}\left(\frac{7}{11}\right)\right)$

42. $\tan\left(\sin^{-1}\left(\frac{9}{13}\right)\right)$

43. ***Chattanooga Temperatures*** The average monthly temperatures in Chattanooga, TN, range from a low of 40.5°F in January to a high of 80.0°F in July. Setting January as month 0 and December as month 11, the temperature cycle can be nicely modeled by a sinusoid with equation $y = A\cos(Bx) + C$, as shown in the graph below. Find the values of A, B, and C. [*Source: www.weatherbase.com*]



44. ***Rocky Mountain Highs*** The average monthly high temperatures in Steamboat Springs, CO, are shown in Table 1.6 below. Setting January as month 0 and December as month 11, construct a sinusoid with equation $y = A\cos(Bx) + C$ that models the temperature cycle in Steamboat Springs. Support your answer with a graph and a scatter plot on your calculator. [*Source: www.weatherbase.com*]

TABLE 1.6 Average Monthly Highs in Steamboat Springs, CO

JAN	FEB	MAR	APR	MAY	JUN
28.9	33.8	42.0	53.6	65.4	75.5
JUL	AUG	SEP	OCT	NOV	DEC
82.6	80.3	72.5	60.4	43.3	30.7

45. ***Even-Odd***

(a) Show that $\cot x$ is an odd function of x.

(b) Show that the quotient of an even function and an odd function is an odd function.

46. ***Even-Odd***

(a) Show that $\csc x$ is an odd function of x.

(b) Show that the reciprocal of an odd function is odd.

47. ***Even-Odd*** Show that the product of an even function and an odd function is an odd function.

48. ***Finding the Period*** Give a convincing argument that the period of $\tan x$ is π.

49. ***Is the Product of Sinusoids a Sinusoid?*** Make a conjecture, and then use your graphing calculator to support your answers to the following questions.

(a) Is the product $y = (\sin x)(\sin 2x)$ a sinusoid? What is the period of the function?

(b) Is the product $y = (\sin x)(\cos x)$ a sinusoid? What is the period of the function?

(c) One of the functions in (a) or (b) above can be written in the form $y = A\sin(Bx)$. Identify the function and find A and B.

Standardized Test Questions

You may use a graphing calculator to solve the following problems.

50. True or False The period of $y = \sin(x/2)$ is π. Justify your answer.

51. True or False The amplitude of $y = \frac{1}{2}\cos x$ is 1. Justify your answer.

In Exercises 52–54, $f(x) = 2\cos(4x + \pi) - 1$.

52. Multiple Choice Which of the following is the domain of f?

(A) $[-\pi, \pi]$ (B) $[-3, 1]$ (C) $[-1, 4]$ (D) $(-\infty, \infty)$ (E) $x \neq 0$

53. Multiple Choice Which of the following is the range of f?

(A) $(-3, 1)$ (B) $[-3, 1]$ (C) $(-1, 4)$ (D) $[-1, 4]$ (E) $(-\infty, \infty)$

54. Multiple Choice Which of the following is the period of f?

(A) 4π (B) 3π (C) 2π (D) π (E) $\pi/2$

55. Multiple Choice Which of the following is the measure of $\tan^{-1}(-\sqrt{3})$ in degrees?

(A) $-60°$ (B) $-30°$ (C) $30°$ (D) $60°$ (E) $120°$

Exploration

56. ***Trigonometric Identities*** Let $f(x) = \sin x + \cos x$.

(a) Graph $y = f(x)$. Describe the graph.

(b) Use the graph to identify the amplitude, period, horizontal shift, and vertical shift.

(c) Use the formula

$$\sin\alpha\cos\beta + \cos\alpha\sin\beta = \sin(\alpha + \beta)$$

for the sine of the sum of two angles to confirm your answers.

Extending the Ideas

57. Exploration Let $y = \sin(ax) + \cos(ax)$.

Use the symbolic manipulator of a computer algebra system (CAS) to help you with the following:

(a) Express y as a sinusoid for $a = 2, 3, 4$, and 5.

(b) Conjecture another formula for y for a equal to any positive integer n.

(c) Check your conjecture with a CAS.

(d) Use the formula for the sine of the sum of two angles (see Exercise 56c) to confirm your conjecture.

58. Exploration Let $y = a \sin x + b \cos x$.

Use the symbolic manipulator of a computer algebra system (CAS) to help you with the following:

(a) Express y as a sinusoid for the following pairs of values: $a = 2, b = 1$; $a = 1, b = 2$; $a = 5, b = 2$; $a = 2, b = 5$; $a = 3, b = 4$.

(b) Conjecture another formula for y for any pair of positive integers. Try other values if necessary.

(c) Check your conjecture with a CAS.

(d) Use the following formulas for the sine or cosine of a sum or difference of two angles to confirm your conjecture.

$$\sin\alpha\cos\beta \pm \cos\alpha\sin\beta = \sin(\alpha \pm \beta)$$

$$\cos\alpha\cos\beta \pm \sin\alpha\sin\beta = \cos(\alpha \mp \beta)$$

In Exercises 59 and 60, show that the function is periodic and find its period.

59. $y = \sin^3 x$ **60.** $y = |\tan x|$

In Exercises 61 and 62, graph one period of the function.

61. $f(x) = \sin(60x)$ **62.** $f(x) = \cos(60\pi x)$

Quick Quiz for AP* Preparation: Sections 1.4–1.6

1. Multiple Choice Which of the following is the domain of $f(x) = -\log_2(x + 3)$?

(A) $(-\infty, \infty)$ **(B)** $(-\infty, 3)$ **(C)** $(-3, \infty)$
(D) $[-3, \infty)$ **(E)** $(-\infty, 3]$

2. Multiple Choice Which of the following is the range of $f(x) = 5\cos(x + \pi) + 3$?

(A) $(-\infty, \infty)$ **(B)** $[2, 4]$ **(C)** $[-8, 2]$
(D) $[-2, 8]$ **(E)** $\left[-\frac{2}{5}, \frac{8}{5}\right]$

3. Multiple Choice Which of the following gives the solution of $\tan x = -1$ in $\pi < x < \frac{3\pi}{2}$?

(A) $-\frac{\pi}{4}$ **(B)** $\frac{\pi}{4}$ **(C)** $\frac{\pi}{3}$ **(D)** $\frac{3\pi}{4}$ **(E)** $\frac{5\pi}{4}$

4. Free Response Let $f(x) = 5x - 3$.

(a) Find the inverse g of f.

(b) Compute $(f \circ g)(x)$. Show your work.

(c) Compute $(g \circ f)(x)$. Show your work.

CHAPTER 1 Key Terms